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Indirect estimation of bedload flux from modern sand-bed rivers and ancient fluvial strata

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ABSTRACT

Bedload sediment transport is a fundamental parameter controlling geomorphic processes in alluvial rivers. Direct measurement in modern rivers is labor intensive, typically cost prohibitive, and often dangerous. In addition, few reliable methods exist for estimating bedload fluxes from ancient fluvial strata. In this paper, we present a model for estimating bedload transport rates from modern sand-bed fluvial systems and ancient fluvial sandstones using substitutions into the bedform-bedload equation. This is enabled by a newly defined empirical relationship between characteristic bedform migration rate and reach slope in normal-flow reaches. For modern rivers, this slope-velocity relationship is combined with published relationships for bedform scales to allow remote estimation of bedload flux. In stratigraphic applications, the slope-velocity model is combined with relationships for bedform scale from cross-set thickness and for paleoslope from barform height and grain size to enable estimation of ancient bedload. The modern application is evaluated using bedload and bedform data from a single survey in the North Loup River near Taylor, Nebraska, USA. The stratigraphic application is demonstrated using outcrop data from the Jurassic Kayenta Formation in Colorado National Monument, Colorado, USA. These two applications yield approximately order-of-magnitude total uncertainty (95% interval) in mean bedload flux estimates.

INTRODUCTION

Rates of bedload sediment transport are among the primary predictors of geomorphic work in alluvial rivers (Engelund and Hansen, 1967; Church, 2006; Gomez, 2006). Bedload transport rates in rivers vary longitudinally and temporally by many orders of magnitude (Hinton et al., 2017) and can reflect potentially significant shifts in the forcing mechanisms on fluvial systems (Syvitski et al., 2005). Bedload flux can contribute a substantial but widely varying proportion of total bed material flux in sandy rivers (<5% to >50%; e.g., Mohrig and Smith, 1996; Turowski et al., 2010). Quantitative assessment of bedload flux is therefore a critical component in understanding alluvial river dynamics, both in modern and ancient environments.

Long-term monitoring of bedload flux is considered important in many river management programs (e.g., Gray et al., 2010). However, the ability to measure bedload flux directly in modern rivers is often hindered by the considerable labor and costs required to collect point measurements. This limits the availability of data for most rivers over varying hydrographs (Gomez, 2006; Gray et al., 2010). As such, methods for calculating bedload flux from shear stress and bed material composition are commonplace (Meyer-Peter and Müller, 1948; Einstein, 1950; van Rijn, 1984; Wong and Parker, 2006). However, these methods require partitioning of total stress exerted on the bed, and quantifying grain stress responsible for particle motion (e.g., Nittrouer et al., 2012) based on either quantitative or qualitative descriptions of bed topography and river planform geometry. Bedload flux can also be calculated using well-developed geometric methods (after Simons et al., 1965) from observations of bedform scale and velocity in sand-bed rivers where bedforms are observable (see Fig. 1A). However, measurements of bedform evolution require repeat bathymetric surveys, which can be costly and logistically difficult in remote locations.

As in modern rivers, quantitative determination of flow and sediment transport conditions from ancient river deposits can yield critical insight into process changes, both across space (outcrop to outcrop across a basin or between contemporaneous basins) and through time (vertical section). Bedform and barform cross stratification are commonly preserved in fluvial strata (see Figs. 1B and 1C). These architectures can provide insight into the relevant scales of topographic features in past environments, such as dune heights and flow depths. These scales may then be used in conjunction with information

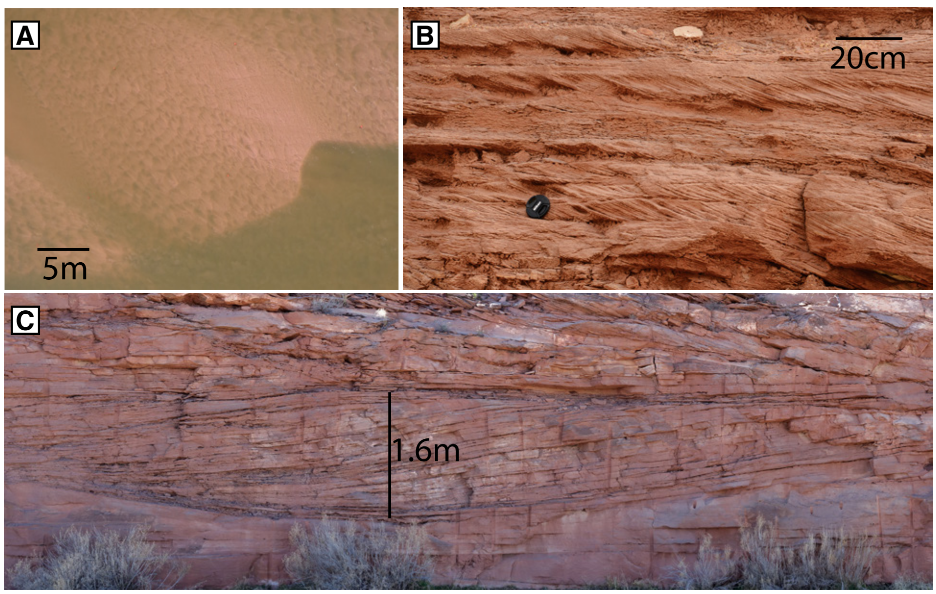

Figure 1. A: Dunes migrating over bars on Niobrara River near Norden, Nebraska, USA (aerial view, flow and transport from upper left to lower right, darker greens are deeper water; 42.787559°N, 100.030014°W). B: Bedform cross strata in Kayenta Formation near Paradox, Colorado, USA (38.325026°N, 108.984445°W). C: Bar clinoforms in Kayenta Formation in Colorado National Monument (39.108729°N, 108.737733°W).

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about the kinematics of bedform migration to determine bedload flux. In ancient systems, however, methods to quantify rates of sediment transport remain largely unconstrained. Methods based on bed shear stress developed in modern rivers have been applied in stratigraphic inversion, although rigorous treatment of uncertainty in the application of these models in the inverse is not commonly done (e.g., Jones and Frostick, 2008).

In this paper, we present a model for estimating bedload transport rates from modern sand-bed rivers with mobile bedforms and from ancient fluvial sandstones. We present a new reach slope–based bedform-velocity relationship, enabling substitutions into the bedform bedload equation of Simons et al. (1965). In modern-river applications, this reach slope–bedform velocity relationship is combined with measurements of bedform length to compute bedload flux. In stratigraphic applications, bedload flux can be calculated from outcrop measurements of bed-set thicknesses, bar heights, and median bed material grain diameter. We test this modern model using previously published direct measurements of bedload flux from the North Loup River near Taylor, Nebraska, USA (Mohrig and Smith, 1996). Stratigraphic applicability is demonstrated using outcrop measurements of the Jurassic Kayenta Formation, a fluvial sandstone exposed across the Colorado Plateau region (USA). In each case, model and measurement uncertainty is directly accounted for and propagated through the model system.

MODEL

The primary thesis of this paper is that unit bedload flux (volume of bedload sediment [L^3] per unit width [L^{-1}] per unit time [T]), q_b (dimensions L^2/T), can be determined from remotely sensed rivers and from fluvial bedform deposits by coupling estimates of bedform geometry and of reach-scale longitudinal channel slope within the bedform-bedload equation of Simons et al. (1965) by substituting semi-empirical relationships for bedform height, h_d, and characteristic migration velocity, V_c, into their formula:

$$q_b = (1-\varphi)\frac{h_d V_c}{2}, \qquad (1)$$

where φ represents dimensionless bed porosity. Bedload is explicitly defined, therefore, as the component of bed material load traveling over distances shorter than the lengths of bedforms and not bypassing lee faces.

This formula has been extensively applied to the calculation of bedload from repeat single- and multi-beam bathymetric surveys of migrating bedforms (e.g., Gaeuman and Jacobson, 2006; Nittrouer et al., 2008). However, the time and expense required to collect these

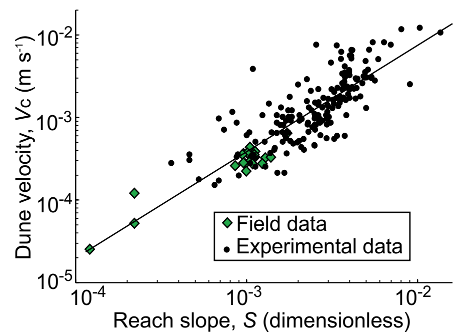

Figure 2. Characteristic migration velocity of river dunes as function of reach slope. Data compilation modified from Lin and Venditti (2013).

types of surveys limit their availability as well as their spatial and temporal extents and resolutions. Additional methods to generate estimates of bedform heights and velocities could widely expand the applicability of the bedform-bedload equation (Equation 1). The most important step in realizing this potential is a model for estimating the kinematics of bedforms (migration velocity) from readily available parameters.

We propose a model for bedform migration velocity from reach-averaged longitudinal channel slope, S (–, dimensionless), in steady, uniform, normal-flow reaches. Using data compiled by Lin and Venditti (2013; see Fig. 2, and data and sources in the GSA Data Repository[1]), we find the following power-law function describing this relationship:

$$\log V_c = \beta_0 + \beta_1 \log S, \qquad (2)$$

where β_0 and β_1 are empirical coefficients whose values and uncertainties are reported in Table 1.

This model is based on a data set comprising 201 independent observations from natural rivers and flume experiments of migrating sandy bedforms. It is important to note that the data set includes only normal-flow hydraulic conditions. We observe that this relationship does not hold in backwater-influenced reaches. Potentially, friction slope is an appropriate model parameter for those locales. However, insufficient data exist to evaluate that hypothesis at present.

Significant variation of dune migration rate about the slope-based model is apparent. The reasons for the variability likely relate to flow strength (e.g., river stage; Gabel, 1993). However, the addition of parameters such as depth, grain size, settling velocity, Froude number, flow velocity, transport stage, and others fails to significantly improve the predictive power of the exclusively slope-dependent model presented in Equation 2.

A velocity model based on reach slope alone has several significant advantages in both remotely sensed modern applications and stratigraphic applications. Reach slope can be determined from local high-precision GPS measurements of water surface elevations or from digital elevation models (DEMs) over longer reaches. For stratigraphic applications, reach slope can be inverted from a variety of methods. The assumption of normal flow required in the use of this model implies independence of slope from flow stage, and thus a single reach value can be applied across a hydrograph, implying that changes in bedform scale are largely responsible for variations in flux.

Model for Modern Rivers

An estimate of bedform height must accompany migration velocity in order to apply the bedform-bedload equation in modern rivers. In some modern cases, this would be directly obtained from single bathymetric surveys. In other remotely sensed scenarios (e.g., aerial imagery), bedform length is potentially the only observable parameter. In this case, a model relating bedform length, L_b to bedform height is applied following Bradley and Venditti (2017) as:

$$h_d = \delta_0 L_b^{\delta_1}, \qquad (3)$$

where δ_0 and δ_1 are empirical parameters with associated uncertainties (see Table 1).

Model for Fluvial Strata

To develop the stratigraphic inversion of bedload, we incorporated a paleoslope reconstruction from outcrop-measurable quantities. Many methods presently exist for this (Paola and Mohrig, 1996; Lynds et al., 2014). We employ the model of Trampush et al. (2014) relating reach slope to bankfull depth, H_{bf}, and median grain size, D_{50}:

$$\log S = \alpha_0 + \alpha_1 \log D_{50} + \alpha_2 \log H_{bf}, \qquad (4)$$

where α_0, α_1, and α_2 are empirical parameters with associated uncertainties (see Table 1). Grain size is determined directly from outcrop observation, and bankfull depth can be estimated from

TABLE 1. MODEL PARAMETERS AND ASSOCIATED UNCERTAINTIES FOR EQUATIONS 2–5

	α_0	α_1	α_2	β_0	β_1	γ	δ_0	δ_1
Coefficient	–2.08	0.254	–1.09	0.6113	1.305	2.9	0.0513	0.7744
Standard deviation	0.036	0.016	0.044	0.144	0.0515	0.7	0.0017	0.0123

Note: Sources of parameter and uncertainty values are referenced in text.

[1]GSA Data Repository item 2018193, bedform velocity data and Matlab functions for simulating bedload fluxes, is available online at http://www.geosociety.org/datarepository/2018/ or on request from editing@geosociety.org.

measurements of bar clinoform heights (Mohrig et al., 2000). Empirical paleoslope reconstructions, such as the one applied here, typically involve approximately an order-of-magnitude variation about fitted models. Ultimately, this is a significant control on the uncertainty of any quantitative estimation of past flow conditions, as is the case in the bedload flux model developed here.

A model for bedform height must be included because height is rarely directly observable in the stratigraphic record. Typically, the tops of bedforms are truncated by antecedent scour troughs. A relationship for the determination of dune height from mean bed-set thickness, t_{sm}, (after Leclair and Bridge, 2001) is included in the inversion

$$H_d = \gamma t_{sm}, \quad (5)$$

where γ is an empirical coefficient with associated uncertainty (see Table 1).

Model Implementation

Different equation sets are used when predicting bedload flux from modern rivers and ancient strata. In modern-river applications where reach slope and bedform heights are known, Equations 1 and 2 are used. Where bedform lengths are known, as in the case where aerial images of bedform fields are available, Equations 1–3 are used. For stratigraphic applications in which truncated bedform cross sets and bar clinoforms are measurable from outcrop, Equations 1, 2, 4, and 5 are used. Values for each parameter and coefficient in Equations 2–5 were empirically determined and are presented along with their associated uncertainty in Table 1. Uncertainty values for δ_0 and δ_1 (after Bradley and Venditti, 2017) were not included in the original publication but were provided by R.W. Bradley (2017, personal commun.).

In order to estimate uncertainty in the combined model outcomes, we assume that uncertainties in model parameters are normally distributed. Monte Carlo simulations are conducted wherein 10^7 iterations of randomly sampled values of each parameter and coefficient are used to calculate individual bedload flux values. This approach assumes that the error values of separate model parameters do not co-vary. Field- and outcrop-derived uncertainty is also incorporated by simultaneous, random sampling of values from each measured array of field parameters (i.e., bootstrapping with replacement). The model output is a distribution of mean bedload flux estimates. Complete model codes for both modern and stratigraphic applications are available as Matlab functions in the Data Repository. Model evaluation is conducted using a linear approach so that evaluation is conducted relatively rapidly over large numbers of iterations (order seconds).

APPLICATION AND DISCUSSION

Modern River Application

The model presented in this paper provides the opportunity to assess bedload flux from modern rivers where direct bedload monitoring may prove too costly to be reasonably conducted. To demonstrate the applicability, and to test the proposed model, we compare model estimates of bedload flux over the back of mid-channel bars in the North Loup River from direct measurements reported by Mohrig and Smith (1996). The applied equation set for this purpose comprises Equations 1–3. Channel slope was surveyed as 1.37×10^{-3}. Bedform lengths were extracted from aerial imagery collected concurrently with bedload measurements. Both the imagery as well as the measured bedload samples cannot be assumed to represent the entire reach-scale variability of depths, and some bias toward shallower depths is likely in both cases. Although we cannot quantify this bias, we can compare the overall ranges of depths from which flux measurements were presented by Mohrig and Smith (1996).

Model results are presented in Figure 3 alongside bedload sample measurements. Unit width bedload flux (q_b) from measured data ranged from 1.0×10^{-5} m² s⁻¹ to 7.9×10^{-5} m² s⁻¹, as compared with 0.02 and 0.98 quantiles of modeled bedload flux of 6.7×10^{-6} m² s⁻¹ and 6.3×10^{-5} m² s⁻¹, respectively. Median values of measured and modeled data were found to be 4.1×10^{-5} and 2.1×10^{-5} m² s⁻¹, respectively.

Given the generally large uncertainties involved in any measurement or calculation of bedload fluxes, we find these results to be very encouraging. The model uncertainty range spans approximately the same order of magnitude as the error range of individual, direct bedload flux measurements taken in the field. Although this is not a rigorous test (i.e., there is only one observation set modeled), it appears as though our model is capable of producing high-quality estimates of bedload flux distributions. Additionally, for cases where bedform height is known, this model may be used without the uncertainty derived from transforming bedform lengths to heights. This may be useful in cases where either non-repeated bathymetric profiles are collected or fixed-position depth profilers are deployed (e.g., at a bridge pier).

Stratigraphic Application

The model presented in this paper provides potential for expanding interpretations of ancient depositional systems in a quantitative framework. Placing reasonable bounds on bedload flux from fluvial outcrops could yield insight into basin-scale trends in fluvial dynamics and associated basin-scale mass conservation. This can be accomplished in both temporal and spatial frameworks. We illustrate this with calculations of bedload flux using measurements of bar clinoforms, sediment grain sizes, and cross-set thicknesses from outcrops of a well-exposed ancient river system, the Jurassic Kayenta Formation in Colorado National Monument in western Colorado.

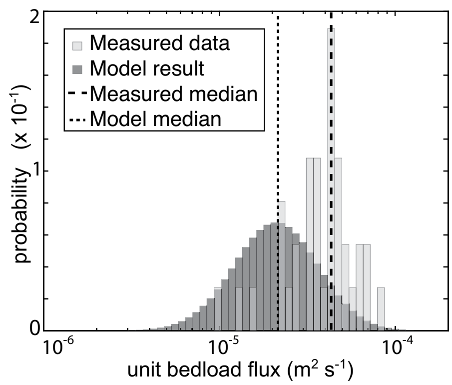

Figure 3. Histograms of measured (light gray) versus modeled (dark gray) unit bedload flux values for North Loup River near Taylor, Nebraska, USA. Model uses bedform lengths, reach slope, and bedload comparison data from Mohrig and Smith (1996).

Bar clinoform heights were assumed to reflect an approximation for bankfull flow depth and were used in conjunction with field determinations of median grain diameter (~300 μm) to calculate paleoslope (after Trampush et al., 2014). Due to the limited number of field measurements of complete bar clinoforms, the resultant paleoslope estimate is bimodal (Fig. 4A). The distribution of paleoslope estimates were then sampled along with measured bedform cross-set thicknesses and used to determined unit bedload flux (Fig. 4B). The estimated values of mean bedload flux from the Kayenta Formation at Colorado National Monument, as captured by the 2% and 98% quantiles, range from 4.1×10^{-6} to 1.2×10^{-4} m² s⁻¹, respectively, with a median calculated flux of 2.2×10^{-5} m² s⁻¹.

Application of this model to the Kayenta Formation demonstrates that, given the limitations

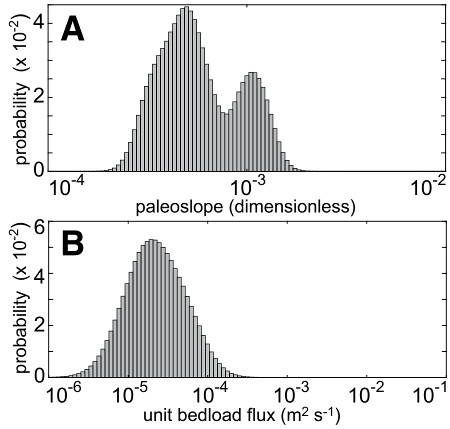

Figure 4. Histograms of estimated paleoslope (A) and estimated mean bedload flux (B) from Jurassic Kayenta Formation in Colorado National Monument, Colorado, USA.

of measurable stratigraphic parameters, it may be reasonable to determine ancient bedload fluxes with approximately order-of-magnitude uncertainty. When taken in conjunction with measurements over regional scales, or through stratigraphic time, it may be possible to interpret changes in the dynamics of sediment transport in a quantitative framework, and place further constraints on transport systems by informing evaluations based on sediment mass conservation (e.g., Paola et al., 1992).

CONCLUSION

Bedload fluxes in sand-bed rivers can be appropriately estimated using a series of empirical models for bedform migration rate and bedform geometry applied to the bedform-bedload equation. This model employs a novel, empirical relationship between bedform migration and reach-scale longitudinal channel slope in normal-flow reaches. In conjunction with previously published relationships, this model can be used to evaluate bedload transport rates from both remotely sensed modern rivers and ancient fluvial strata. In remotely sensed rivers, bedform length is inverted from aerial imagery to give bedform height and is combined with DEM-based channel slope. In ancient fluvial strata, bed-set thicknesses are inverted to give bedform height and are combined with paleoslope estimates that derive from bar clinoform height, a proxy for bankfull depth, and grain diameter. Both the modern and ancient models can be implemented using a Monte Carlo analysis to honor measurement uncertainty as well as intrinsic, physical parameter variability in rivers.

The modern-river application was tested against measurements from the North Loup River near Taylor, Nebraska. Model uncertainty ranges are in good agreement with measured values. The stratigraphic model is demonstrated using measurements from the Jurassic Kayenta Formation in Colorado National Monument. Results of this inversion show uncertainty slightly greater than one order of magnitude, derived from the model system and measurement variability. Taken together, these two models are a novel approach to the determination of bedload flux from modern rivers, where flux measurements are sparse, as well as from ancient fluvial strata, where fewer constraints on bedload flux exist. Application of this latter model to stratigraphic problems offers the potential for greatly improved understanding of surface process dynamics in ancient sedimentary systems over a broad range of spatial and temporal scales.

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